ANSWERS

1.	**3**	16.	**2**	31.	**4**	46.	**4**	61.	**1**
2.	**1**	17.	**1**	32.	**2**	47.	**3**	62.	**3**
3.	**3**	18.	**1**	33.	**1**	48.	**4**	63.	**3**
4.	**2**	19.	**2**	34.	**1**	49.	**2**	64.	**4**
5.	**4**	20.	**2**	35.	**2**	50.	**1**	65.	**3**
6.	**2**	21.	**1**	36.	**2**	51.	**1**	66.	**3**
7.	**2**	22.	**4**	37.	**4**	52.	**1**	67.	**3**
8.	**1**	23.	**3**	38.	**1**	53.	**1**	68.	**2**
9.	**4**	24.	**1**	39.	**3**	54.	**2**	69.	**4**
10.	**3**	25.	**4**	40.	**2**	55.	**3**	70.	**3**
11.	**4**	26.	**2**	41.	**1**	56.	**3**	71.	**3**
12.	**3**	27.	**2**	42.	**1**	57.	**2**	72.	**1**
13.	**4**	28.	**2**	43.	**3**	58.	**1**	73.	**1**
14.	**1**	29.	**3**	44.	**2**	59.	**4**	74.	**4**
15.	**3**	30.	**3**	45.	**1**	60.	**4**	75.	**2**

SOLUTIONS

1. (2, 3) $\in$ R but (3, 2) $\notin$ R.
Hence R is not symmetric.

2. $f(x) = {}^{7-x}P_{x-3}$
$7 - x \geq 0 \quad \Rightarrow \quad x \leq 7$
$x - 3 \geq 0 \quad \Rightarrow \quad x \geq 3$,
and $7 - x \geq x - 3 \quad \Rightarrow \quad x \leq 5$
$\Rightarrow 3 \leq x \leq 5 \Rightarrow x = 3, 4, 5 \Rightarrow$ Range is {1, 2, 3}.

3. Here $\omega = \dfrac{z}{i} \Rightarrow \arg\left(z.\dfrac{z}{i}\right) = \pi \Rightarrow 2\arg(z) - \arg(i) = \pi \Rightarrow \arg(z) = \dfrac{3\pi}{4}$.

4. $z = (p + iq)^3 = p\left(p^2 - 3q^2\right) - iq\left(q^2 - 3p^2\right)$

$$\Rightarrow \frac{x}{p} = p^2 - 3q^2 \;\; \& \;\; \frac{y}{q} = q^2 - 3p^2 \Rightarrow \frac{\frac{x}{p} + \frac{y}{q}}{\left(p^2 + q^2\right)} = -2.$$

5. $\left|z^2 - 1\right|^2 = \left(|z|^2 + 1\right)^2 \Rightarrow \left(z^2 - 1\right)\left(\bar{z}^2 - 1\right) = |z|^4 + 2|z|^2 + 1$
$\Rightarrow z^2 + \bar{z}^2 + 2z\bar{z} = 0 \Rightarrow z + \bar{z} = 0$
$\Rightarrow$ R (z) = 0 $\Rightarrow$ z lies on the imaginary axis.

6. $A.A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$.

7. AB = I $\Rightarrow$ A(10 B) = 10 I

$$\Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{if } \alpha = 5.$$

8. $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$C_3 \to C_3 - C_2$, $C_2 \to C_3 - C_1$

$$= \begin{vmatrix} \log a_n & \log r & \log r \\ \log a_{n+3} & \log r & \log r \\ \log a_{n+6} & \log r & \log r \end{vmatrix} = 0 \quad \text{(where r is a common ratio).}$$

9. Let numbers be a, b $\Rightarrow a + b = 18$, $\sqrt{ab} = 4 \Rightarrow ab = 16$, a and b are roots of the equation

$\Rightarrow \quad x^2 - 18x + 16 = 0$.

10. **(3)**

$(1-p)^2 + p(1-p) + (1-p) = 0$ (since (1 – p) is a root of the equation $x^2 + px + (1 - p) = 0$)

$\Rightarrow \quad (1-p)(1-p+p+1) = 0$

$\Rightarrow \quad 2(1-p) = 0 \Rightarrow (1 - p) = 0 \Rightarrow p = 1$

sum of root is $\alpha + \beta = -p$ and product $\alpha\beta = 1 - p = 0$ (where $\beta = 1 - p = 0$)

$\Rightarrow \quad \alpha + 0 = -1 \quad \Rightarrow \quad \alpha = -1 \Rightarrow$ Roots are 0, –1

11. $S(k) = 1 + 3 + 5 + ... + (2k-1) = 3 + k^2$

S(k + 1)=1 + 3 + 5 +............ + (2k – 1) + (2k + 1)

$= (3 + k^2) + 2k + 1 = k^2 + 2k + 4$ [from $S(k) = 3 + k^2$]

$= 3 + (k^2 + 2k + 1) = 3 + (k + 1)^2 = S(k + 1)$.

Although S (k) in itself is not true but it considered true will always imply towards S (k + 1).

12. Since in half the arrangement A will be before E and other half E will be before A.

Hence total number of ways = $\frac{6!}{2}$ = 360.

13. Number of balls = 8

number of boxes = 3

Hence number of ways = ${}^7C_2 = 21$.

14. Since 4 is one of the root of $x^2 + px + 12 = 0 \Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

and equation $x^2 + px + q = 0$ has equal roots

$\Rightarrow D = 49 - 4q = 0 \Rightarrow q = \frac{49}{4}$.

15. Coefficient of Middle term in $(1 + \alpha x)^4 = t_3 = {}^4C_2 \cdot \alpha^2$

Coefficient of Middle term in $(1 - \alpha x)^6 = t_4 = {}^6C_3 (-\alpha)^3$

${}^4C_2\alpha^2 = -{}^6C_3.\alpha^3 \Rightarrow \quad -6 = 20\alpha \quad \Rightarrow \quad \alpha = \frac{-3}{10}$

16. Coefficient of x^n in $(1 + x)(1 - x)^n = (1 + x)({}^nC_0 - {}^nC_1x + ... + (-1)^{n-1}\,{}^nC_{n-1}\,x^{n-1} + (-1)^n\,{}^nC_n\,x^n)$

$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_{n-1} = (-1)^n (1-n)$.

17. $t = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r} \quad (\because\ {}^nC_r = {}^nC_{n-r})$

$2t_n = \sum_{r=0}^{n} \frac{r+n-r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n}{{}^nC_r} \Rightarrow \quad t_n = \frac{n}{2}\sum_{r=0}^{n} \frac{1}{{}^nC_r} = \frac{n}{2}S_n \quad \Rightarrow \quad \frac{t_n}{S_n} = \frac{n}{2}$

18. $T_m = \frac{1}{n} = a + (m-1)d$ .....(1)

and $T_n = \frac{1}{m} = a + (n-1)d$ .....(2)

from (1) and (2) we get $a = \frac{1}{mn}, \quad d = \frac{1}{mn}$

Hence a – d = 0

19. If n is odd then (n – 1) is even $\Rightarrow$ sum of odd terms $= \frac{(n-1)n^2}{2} + n^2 = \frac{n^2(n+1)}{2}$.

20. $\frac{e^{\alpha} + e^{-\alpha}}{2} = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots\ldots$

$\frac{e^{\alpha} + e^{-\alpha}}{2} - 1 = \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \frac{\alpha^6}{6!} + \ldots\ldots.$

put α = 1, we get

$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots\ldots\ldots$

21. $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$.

Squaring and adding, we get

$2 + 2\cos(\alpha - \beta) = \frac{1170}{(65)^2}$

$\Rightarrow \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{130} \Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}} \qquad \left(\because \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}\right)$.

22. $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$= \sqrt{\frac{a^2+b^2}{2} + \frac{a^2-b^2}{2}\cos 2\theta} + \sqrt{\frac{a^2+b^2}{2} + \frac{b^2-a^2}{2}\cos 2\theta}$

$\Rightarrow u^2 = a^2 + b^2 + 2\sqrt{\left(\frac{a^2+b^2}{2}\right)^2 - \left(\frac{a^2-b^2}{2}\right)^2\cos^2 2\theta}$

min value of $u^2 = a^2 + b^2 + 2ab$

max value of $u^2 = 2\left(a^2 + b^2\right)$

$\Rightarrow u^2_{max} - u^2_{min} = (a-b)^2$.

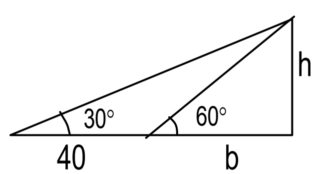
23. Greatest side is $\sqrt{1 + \sin\alpha\cos\alpha}$, by applying cos rule we get greatest angle = 120°.

24. $\tan 30° = \frac{h}{40+b}$

$\Rightarrow \sqrt{3}h = 40 + b$ .....(1)

$\tan 60° = h/b \Rightarrow h = \sqrt{3}\, b$ ....(2)

$\Rightarrow$ b = 20 m



25. $-2 \le \sin x - \sqrt{3}\cos x \le 2 \Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$

$\Rightarrow$ range of f(x) is [–1, 3].

Hence S is [–1, 3].

26. If y = f (x) is symmetric about the line x = 2 then f(2 + x) = f(2 – x).

27. $9 - x^2 > 0$ and $-1 \le x - 3 \le 1 \Rightarrow x \in [2, 3)$

28. $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x} = \lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)\times 2x\times\left(\frac{a}{x}+\frac{b}{x^2}\right)} = e^{2a} \Rightarrow a = 1,\ b \in R$

29. $f(x) = \frac{1-\tan x}{4x-\pi} \Rightarrow \lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{4x-\pi} = -\frac{1}{2}$

30. $x = e^{y+e^{y+e^{y+.....\infty}}} \Rightarrow x = e^{y+x}$

$\Rightarrow \ln x - x = y \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$.

31. Any point be $\left(\frac{9}{2}t^2,\ 9t\right)$; differentiating $y^2 = 18x$

$\Rightarrow \frac{dy}{dx} = \frac{9}{y} = \frac{1}{t} = 2$ (given) $\Rightarrow t = \frac{1}{2}$.

$\Rightarrow$ Point is $\left(\frac{9}{8},\ \frac{9}{2}\right)$

32. $f''(x) = 6(x-1) \Rightarrow f'(x) = 3(x-1)^2 + c$
and $f'(2) = 3 \Rightarrow c = 0$
$\Rightarrow f(x) = (x-1)^3 + k$ and $f(2) = 1 \Rightarrow k = 0$
$\Rightarrow f(x) = (x-1)^3$.

33. Eliminating θ, we get $(x-a)^2 + y^2 = a^2$.
Hence normal always pass through (a, 0).

34. Let $f'(x) = ax^2 + bx + c \Rightarrow f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$

$\Rightarrow f(x) = \frac{1}{6}\left(2ax^3 + 3bx^2 + 6cx + 6d\right)$, Now f(1) = f(0) = d, then according to Rolle's theorem

$\Rightarrow f'(x) = ax^2 + bx + c = 0$ has at least one root in (0, 1)

35. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}} = \int_0^1 e^x dx = (e-1)$

36. Put $x - \alpha = t$

$\Rightarrow \int\frac{\sin(\alpha+t)}{\sin t}dt = \sin\alpha\int\cot t\,dt + \cos\alpha\int dt$

$= \cos\alpha(x-\alpha) + \sin\alpha\ln|\sin t| + c$

$A = \cos\alpha,\ B = \sin\alpha$

37. $\int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}} \int \frac{1}{\cos\left(x + \frac{\pi}{4}\right)} dx = \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx = \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$

38. $\int_{-2}^{-1} (x^2 - 1) dx + \int_{-1}^{1} (1 - x^2) dx + \int_{1}^{3} (x^2 - 1) dx = \left.\frac{x^3}{3} - x\right|_{-2}^{-1} + \left. x - \frac{x^3}{3}\right|_{-1}^{1} + \left.\frac{x^3}{3} - x\right|_{1}^{3} = \frac{28}{3}.$

39. $\int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}} dx = \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = \left|-\cos x + \sin x\right|_0^{\frac{\pi}{2}} = 2.$

40. Let $I = \int_0^{\pi} x f(\sin x) dx = \int_0^{\pi} (\pi - x) f(\sin x) dx = \pi \int_0^{\pi} f(\sin x) dx - I$ (since f (2a – x) = f (x))

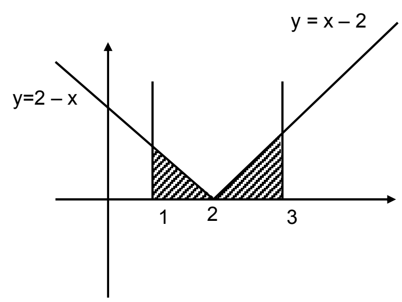
$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x) dx \Rightarrow A = \pi.$

41. f(-a) + f(a) = 1

$I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx = \int_{f(-a)}^{f(a)} (1-x) g\{x(1-x)\} dx \qquad \left(\because \int_a^b f(x) dx = \int_a^b f(a+b-x)\, dx\right)$

$2I_1 = \int_{f(-a)}^{f(a)} g\{x(1-x)\} dx = I_2 \Rightarrow I_2 / I_1 = 2.$

42. Area = $\int_1^2 (2 - x) dx + \int_2^3 (x - 2) dx = 1.$



43. 2x + 2yy′ - 2ay′ = 0

$a = \frac{x + yy'}{y'}$ (eliminating a)

$\Rightarrow (x^2 - y^2)y' = 2xy.$

45. $y\,dx + x\,dy + x^2 y\,dy = 0.$

$\frac{d(xy)}{x^2y^2} + \frac{1}{y} dy = 0 \Rightarrow -\frac{1}{xy} + \log y = C.$

45. If C be (h, k) then centroid is (h/3, (k – 2)/3) it lies on 2x + 3y = 1.

$\Rightarrow$ locus is 2x + 3y = 9.

46. $\frac{x}{a}+\frac{y}{b}=1$ where a + b = -1 and $\frac{4}{a}+\frac{3}{b}=1$

$\Rightarrow$ a = 2, b = -3 or a = -2, b = 1.

Hence $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

47. $m_1 + m_2 = -\frac{2c}{7}$ and $m_1 m_2 = -\frac{1}{7}$

$m_1 + m_2 = 4m_1m_2$ (given)

$\Rightarrow$ c = 2.

48. $m_1 + m_2 = \frac{1}{4c}$, $m_1m_2 = \frac{6}{4c}$ and $m_1 = -\frac{3}{4}$.

Hence c = -3.

49. Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow c = 4$ and it passes through (a, b)

$\Rightarrow a^2 + b^2 + 2ga + 2fb + 4 = 0$.

Hence locus of the centre is $2ax + 2by – (a^2 + b^2 + 4) = 0$.

50. Let the other end of diameter is (h, k) then equation of circle is

$(x – h)(x – p) + (y – k)(y – q) = 0$

Put y = 0, since x-axis touches the circle

$\Rightarrow x^2 – (h + p)x + (hp + kq) = 0 \Rightarrow (h + p)^2 = 4(hp + kq)$ (D = 0)

$\Rightarrow (x – p)^2 = 4qy$.

51. Intersection of given lines is the centre of the circle i.e. (1, – 1)

Circumference = $10\pi \Rightarrow$ radius r = 5

$\Rightarrow$ equation of circle is $x^2 + y^2 – 2x + 2y – 23 = 0$.

52. Points of intersection of line y = x with $x^2 + y^2 – 2x = 0$ are (0, 0) and (1, 1)

hence equation of circle having end points of diameter (0, 0) and (1, 1) is

$x^2 + y^2 – x – y = 0$.

53. Points of intersection of given parabolas are (0, 0) and (4a, 4a)

$\Rightarrow$ equation of line passing through these points is y = x

On comparing this line with the given line 2bx + 3cy + 4d = 0, we get

d = 0 and 2b + 3c = 0 $\Rightarrow (2b + 3c)^2 + d^2 = 0$.

54. Equation of directrix is x = a/e = 4 $\Rightarrow$ a = 2

$b^2 = a^2 (1 – e^2) \Rightarrow b^2 = 3$

Hence equation of ellipse is $3x^2 + 4y^2 = 12$.

55. $l = \cos\theta$, $m = \cos\theta$, $n = \cos\beta$

$\cos^2\theta + \cos^2\theta + \cos^2\beta = 1 \Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\cos^2\theta = 3/5$.

56. Given planes are

$2x + y + 2z – 8 = 0$, $4x + 2y + 4z + 5 = 0 \Rightarrow 2x + y + 2z + 5/2 = 0$

Distance between planes = $\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}} = \frac{|-8-5/2|}{\sqrt{2^2+1^2+2^2}} = \frac{7}{2}$.

57. Any point on the line $\frac{x}{1}=\frac{y+a}{1}=\frac{z}{1}=t_1$ (say) is $(t_1, t_1 - a, t_1)$ and any point on the line $\frac{x+a}{2}=\frac{y}{1}=\frac{z}{1}=t_2$ (say) is $(2t_2 - a, t_2, t_2)$.
Now direction cosine of the lines intersecting the above lines is proportional to $(2t_2 - a - t_1, t_2 - t_1 + a, t_2 - t_1)$.
Hence $2t_2 - a - t_1 = 2k$, $t_2 - t_1 + a = k$ and $t_2 - t_1 = 2k$
On solving these, we get $t_1 = 3a$, $t_2 = a$.
Hence points are (3a, 2a, 3a) and (a, a, a).

58. Given lines $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$ and $\frac{x}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$ are coplanar then plan passing through these lines has normal perpendicular to these lines
$\Rightarrow a - b\lambda + c\lambda = 0$ and $\frac{a}{2}+b-c=0$ (where a, b, c are direction ratios of the normal to the plan)
On solving, we get $\lambda = -2$.

59. Required plane is $S_1 - S_2 = 0$
where $S_1 = x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and
$S_2 = x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$
$\Rightarrow 2x - y - z = 1$.

60. $\left(\vec{a}+2\vec{b}\right)=t_1\vec{c}$ ....(1)
and $\vec{b}+3\vec{c}=t_2\vec{a}$ ....(2)
$(1) - 2\times(2) \Rightarrow \vec{a}\left(1+2t_2\right)+\vec{c}\left(-t_1-6\right)=0 \Rightarrow 1+2t_2 = 0 \Rightarrow t_2 = -1/2$ & $t_1 = -6$.
Since $\vec{a}$ and $\vec{c}$ are non-collinear.
Putting the value of t_1 and t_2 in (1) and (2), we get $\vec{a}+2\vec{b}+6\vec{c}=\vec{0}$.

61. Work done by the forces $\vec{F}_1$ and $\vec{F}_2$ is $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$, where $\vec{d}$ is displacement
According to question $\vec{F}_1+\vec{F}_2 = (4\hat{i}+\hat{j}-3\hat{k})+(3\hat{i}+\hat{j}-\hat{k})=7\hat{i}+2\hat{j}-4\hat{k}$
and $\vec{d}=(5\hat{i}+4\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=4\hat{i}+2\hat{j}-2\hat{k}$. Hence $(\vec{F}_1+\vec{F}_2)\cdot\vec{d}$ is 40.

63. Condition for given three vectors to be coplanar is $\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix} = 0 \Rightarrow \lambda = 0, 1/2$.
Hence given vectors will be non coplanar for all real values of λ except 0, 1/2.

63. Projection of $\bar{v}$ along $\bar{u}$ and $\bar{w}$ along $\bar{u}$ is $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}$ and $\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|}$ respectively
According to question $\frac{\bar{v}\cdot\bar{u}}{|\bar{u}|}=\frac{\bar{w}\cdot\bar{u}}{|\bar{u}|} \Rightarrow \bar{v}\cdot\bar{u}=\bar{w}\cdot\bar{u}$. and $\bar{v}\cdot\bar{w}=0$
$|\bar{u}-\bar{v}+\bar{w}|^2 = |\bar{u}|^2 + |\bar{v}|^2 + |\bar{w}|^2 - 2\bar{u}\cdot\bar{v} + 2\bar{u}\cdot\bar{w} - 2\bar{v}\cdot\bar{w} = 14 \Rightarrow |\bar{u}-\bar{v}+\bar{w}| = \sqrt{14}$.

64. $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a} \Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{b}\cdot\vec{c}\right)\vec{a} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \left(\vec{a}\cdot\vec{c}\right)\vec{b} = \left(\frac{1}{3}|\vec{b}||\vec{c}| + \left(\vec{b}\cdot\vec{c}\right)\right)\vec{a} \Rightarrow \vec{a}\cdot\vec{c} = 0$ and $\frac{1}{3}|\vec{b}||\vec{c}| + \left(\vec{b}\cdot\vec{c}\right) = 0$

$\Rightarrow |\vec{b}||\vec{c}|\left(\frac{1}{3} + \cos\theta\right) = 0 \Rightarrow \cos\theta = -1/3 \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$.

65. Mode can be computed from histogram and median is dependent on the scale.
Hence statement (a) and (b) are correct.

66. $x_i = a$ for $i = 1, 2, \ldots, n$ and $x_i = -a$ for $i = n, \ldots, 2n$

S.D. = $\sqrt{\frac{1}{2n}\sum_{i=1}^{2n}(x_i - \bar{x})^2} \Rightarrow 2 = \sqrt{\frac{1}{2n}\sum_{i=1}^{2n}x_i^2}$ $\left(\text{Since } \sum_{i=1}^{2n}x_i = 0\right) \Rightarrow 2 = \sqrt{\frac{1}{2n}\cdot 2na^2} \Rightarrow |a| = 2$

67. E_1 : event denoting that A speaks truth

E_2 : event denoting that B speaks truth

Probability that both contradicts each other = $P\left(E_1 \cap \bar{E}_2\right) + P\left(\bar{E}_1 \cap E_2\right) = \frac{4}{5}\cdot\frac{1}{4} + \frac{1}{5}\cdot\frac{3}{4} = \frac{7}{20}$

68. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$

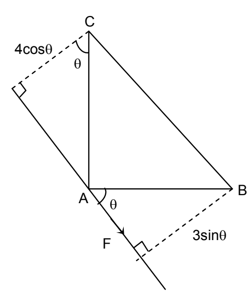
69. Given that $np = 4$, $npq = 2 \Rightarrow q = 1/2 \Rightarrow p = 1/2$, $n = 8 \Rightarrow p(x = 2) = {}^8C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = \frac{28}{256}$

70. $P + Q = 4$, $P^2 + Q^2 = 9 \Rightarrow P = \left(2 + \frac{1}{2}\sqrt{2}\right)$N and $Q = \left(2 - \frac{1}{2}\sqrt{2}\right)$N.

71. $F \cdot 3\sin\theta = 9$

$F \cdot 4\cos\theta = 16$

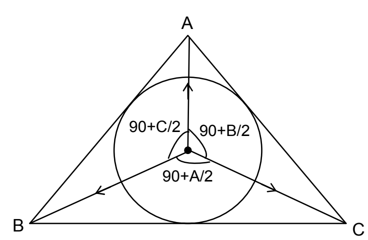
$\Rightarrow F = 5$.



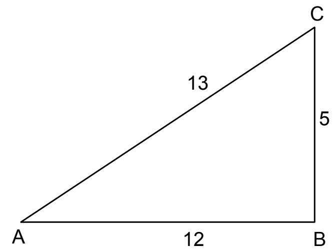
72. By Lami's theorem

$\vec{P} : \vec{Q} : \vec{R} = \sin\left(90^\circ + \frac{A}{2}\right) : \sin\left(90^\circ + \frac{B}{2}\right) : \sin\left(90^\circ + \frac{C}{2}\right)$

$\Rightarrow \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$.



73. Time T_1 from A to B = $\frac{12}{4}$ = 3 hrs.



T_2 from B to C = $\frac{5}{5}$ = 1 hrs.

Total time = 4 hrs.

Average speed = $\frac{17}{4}$ km/ hr.

Resultant average velocity = $\frac{13}{4}$ km/hr.

74. Component along OB = $\frac{\frac{1}{4}\sin 30°}{\sin(45° + 30°)} = \frac{1}{8}\left(\sqrt{6} - \sqrt{2}\right)$ m/s.

75. $t_1 = \frac{2u\sin\alpha}{g}$, $t_2 = \frac{2u\sin\beta}{g}$ where $\alpha + \beta = 90^0$

$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$.